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NUMERICAL SIMULATION OF 3D UNSTEADY VISCOUS FLOW THROUGH RECIPROCALLY MOVING BLADE ROWS

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Abstract

The paper presents results of the numerical investigation of the periodically unsteady flow in the turbojet engine turbine stage with cooling air injection through the stator blade trailing edge. The 3D viscous unsteady flow is simulated with the Reynolds-averaged Navier-Stokes equations and two-equation turbulence model. The phenomena of temperature segregation and stator wake transport through the rotor cascade are considered. The importance of the 3D flow model is emphasized for the simulation of these unsteady phenomena.

Nomenclature

Symbols

b	blade chord
D	root diameter
E	flux vector
H	source term
J	Jacobian of transformation of co-ordinates
k	turbulent kinetic energy
l	blade length or arbitrary integer
n	blade number
m	arbitrary integer
p	pressure
Q	conservative variables vector
q_j	heat flux
r	distance to rotation axis
s	blade pitch
t	time
T	temporal period or temperature
u_j	Cartesian velocity components
U^i	contravariant velocity components
x_j	Cartesian co-ordinates x, y, z
z	axial space
δ_{ij}	Kronecker's delta function
$\Delta\varphi$	spatial period
ε	internal energy
φ	angular tangential co-ordinate
λ	heat conductivity
μ	dynamic viscosity
Ω	angular rotation speed
ρ	density
τ_{ij}	tensor of viscous stresses
ψ^i	curvilinear co-ordinates ξ,η,ζ
$\overline{\psi}^i_j$	metric coefficients

Indices

0	zero time
1,2	stator, rotor respectively
i	number of cascade
i,j,k,l	integer upper and lower indices corresponding to curvilinear and Cartesian co-ordinates respectively
m	molecular
t	turbulent

Abbreviations

ENO	essentially non-oscillatory
RANS	Reynolds-averaged Navier-Stokes equations
SST	shear stress transport

Introduction

Real turbomachinery flows are always unsteady that is mostly caused by the aerodynamic interaction of reciprocally moving rows [1]. The unsteadiness generates a number of phenomena, which cannot be considered under traditional steady problem statement. Turbomachinery unsteady flows have been investigated with experimental approach [2–4 and others]. However, experimental study of such subtle and sophisticated process as an unsteady blade row interaction is accompanied with a number of technical difficulties. Therefore, recently numerical simulation based on the integration of gasdynamic equations is more often used [5–8]. The paper presents results of numerical investigation of the periodically unsteady flow in the turbojet engine turbine stage with cooling air injection. The phenomena of temperature segregation and stator wake transport through rotor cascade are considered. The importance of the 3D flow model is emphasized for simulation of the phenomena.

Problem statement

We consider 3D viscous periodically unsteady flow through turbomachine stage.

The assumption that the reciprocal position of blade rows determines completely the instantaneous flow pattern permits us to cut the computational domain to one blade-to-blade passage for each blade row. In this case it is necessary to use the generalized conditions of space-time periodicity of flow [9,10].

We will consider the turbomachine stage (Fig. 1) that consists of two blade rows with blade numbers n_1 and n_2 respectively. The blade rows rotate with constant angular velocities Ω_1 and Ω_2 correspondingly such that $\Omega_1 \neq \Omega_2$. It is convenient to introduce relative angular coordinates $\overline{\varphi}_1$ and $\overline{\varphi}_2$ fixed to both cascades:

$$\overline{\varphi}_1 = \varphi + \Omega_1 (t - t_0); \quad \overline{\varphi}_2 = \varphi + \Omega_2 (t - t_0); \qquad (1)$$

where φ is absolute angular coordinate, t_0 is time corresponding to an arbitrary selected initial reciprocal position of cascades. The flow through reciprocally moving blade rows is temporally periodical, i.e.

$$Q(\overline{\varphi}_i, t) = Q(\overline{\varphi}_i, t + lT_i)\ ;\ T_i = \frac{2\pi}{n_{3-i}\left|\Omega_{3-i} - \Omega_i\right|}\ . \qquad (2)$$

If the blade numbers for both cascades are equal then the stage flow (similar to isolated cascade flow) is spatially periodical, i.e.

$$Q(\overline{\varphi}_i, t) = Q(\overline{\varphi}_i + m\Delta\varphi_i, t)\ ;\ \Delta\varphi_i = \frac{2\pi}{n_i}\ . \qquad (3)$$

Therefore in this case it is possible to use the usual space periodicity condition. However it is typically for turbomachinery stage that $n_1 \neq n_2$ and the spatial period of a stage flow corresponds generally to 2π. Then instead of the usual space periodicity condition it is necessary to use the time-space periodicity condition:

$$Q(\overline{\varphi}_i, t) = Q(\overline{\varphi}_i + m\Delta\varphi_i, t + \Delta t_i)\ ,$$

$$\Delta t_i = \frac{m\Delta\varphi_i}{\left|\Omega_{3-i} - \Omega_i\right|} + lT_i \qquad (4)$$

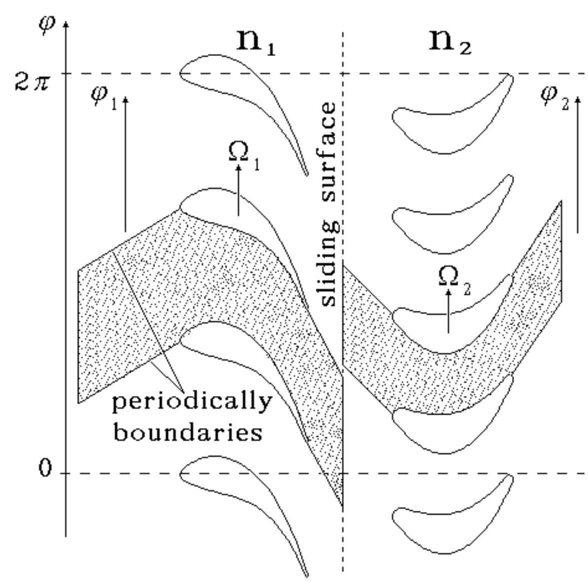


Fig.1: The time-space periodicity condition for turbomachinery stage

The equation (4) means physically that the flow pattern observed at instant t for some blade-to-blade passage of $i-$cascade will be repeated completely for shifted by angle $m\Delta\varphi_i$ passage of this cascade after lapse of time Δt_i that is necessary for reciprocal rotational displacement of rows by angle $m\Delta\varphi_i$. The additional term lT_i that takes into account the time periodicity only (equation (4)) is essential to minimize the time lag Δt_i.

We assume that the flow parameters are known for some blade-to-blade passage of $i-$cascade with angular coordinate

$$\overline{\varphi}_{0i} - \frac{2\pi}{n_i} < \overline{\varphi}_i \le \overline{\varphi}_{0i}\,; \tag{5}$$

at temporal period

$$t_0 - \frac{2\pi}{n_{3-i}\left|\Omega_{3-i} - \Omega_i\right|} < t \le t_0\,. \tag{6}$$

Let it is necessary to find the flow parameters in arbitrary blade-to-blade passage of this cascade for point with angular coordinate $\overline{\varphi}_i^*$ at the instant of time t^*. Applying the relations (4)-(6) to this point yields:

$$\begin{aligned} & t_0 - \frac{2\pi}{n_{3-i}\left|\Omega_{3-i} - \Omega_i\right|} < t = \\ & t^* + \frac{2\pi}{\left|\Omega_{3-i} - \Omega_i\right|}\left(\frac{l}{n_{3-i}} + \frac{m}{n_i}\right) \le t_0 \\ & \overline{\varphi}_{0i} - \frac{2\pi}{n_i} < \overline{\varphi}_i = \overline{\varphi}_i^* + \frac{2\pi m}{n_i} \le \overline{\varphi}_{0i}\,; \end{aligned} \tag{7}$$

From the second inequality (7) we find m and from the first one we define l which are satisfied the condition $q(\overline{\varphi}_i^*, t^*) = q(\overline{\varphi}_i, t)$. So knowing the flow parameters within one blade-to-blade passage of turbomachine stage at temporal period permits the reconstruction of flow pattern in the whole cascade at arbitrary instant of time. Thus we can define the flow parameters outside the computational domain and simulate correctly the influence of neighbouring blade-to-blade passages on the calculated flow. It is necessary practically to store the time dependencies of flow parameters for regions that are located near periodicity boundaries and sliding surface only, not for the whole computational domain.

Method of 3D viscous unsteady flow simulation

The 3D viscous compressible unsteady flow is simulated with a set of unsteady RANS equations written in a curvilinear body-fitted coordinate system, rotating with a constant angular speed:

$$\frac{\partial Q}{\partial t} + \frac{\partial E^i}{\partial \psi_i} = H\,, \tag{8}$$

where

$$Q = J\begin{pmatrix} \rho \\ \rho u_j \\ h \end{pmatrix}; \quad E^i = J\begin{pmatrix} \rho U^i \\ \rho u_j U^i + p\overline{\psi}^i_j - \tau_{jk}\overline{\psi}^i_k \\ (h+p)U^i - (u_l \tau_{kl} - q_k)\overline{\psi}^i_k \end{pmatrix};$$

$$h = \varepsilon + \left(u_l u_l - \Omega^2 r^2\right)/2\,; \quad U^i = u_l \overline{\psi}^i_l\,;$$

$$\overline{\psi}^i_j = \partial\psi^i / \partial x_j\,; \quad \tau_{ij} = 2\mu\left(S_{ij} - S_{ll}\delta_{ij}/3\right) - 2\rho k \delta_{ij}/3;$$

$$\mu = \mu_m + \mu_t\,; \quad q_j = -\lambda \partial T / \partial x_j\,; \quad \lambda = \lambda_m + \lambda_t\,;$$

$$S_{ij} = 0.5\left(\partial u_i / \partial x_j + \partial u_j / \partial x_i\right).$$

Here the summation over repeated indices is performed.

The definition of internal energy ε takes into account the linear dependence of specific heats on temperature.

The simulation of statistic influence of turbulence on the mean flow is performed with the Menter's SST model [11].

The governing equations are numerically integrated by the Godunov's type second-order accurate implicit ENO scheme suggested by Yershov [12].

The code ***FlowER-U***® developed by the authors was used for the flow computations. The problem statement and the numerical approach are described in detail in [9, 13-15].

Test turbine stage

The high-pressure turbine stage of turbojet engine is considered. The geometric data of the turbine stage are given in the Table 1. The stage is characterised by high flow temperature and it requires cooling of blades. The cooling air was injected at the stator trailing edge. The gasdynamic data of the turbine stage are presented in Table 2.

Parameter	Stator	Rotor
Blade number, n	68	94
Relative height, l/b	1.55	2.36
Relative pitch, s/b	0.98	0.87
Root diameter, D [m]	0.633	0.629
Angular rotation speed, Ω [s^{-1}]	0	539.3
Relative axial space, z/l [m]	0.212	

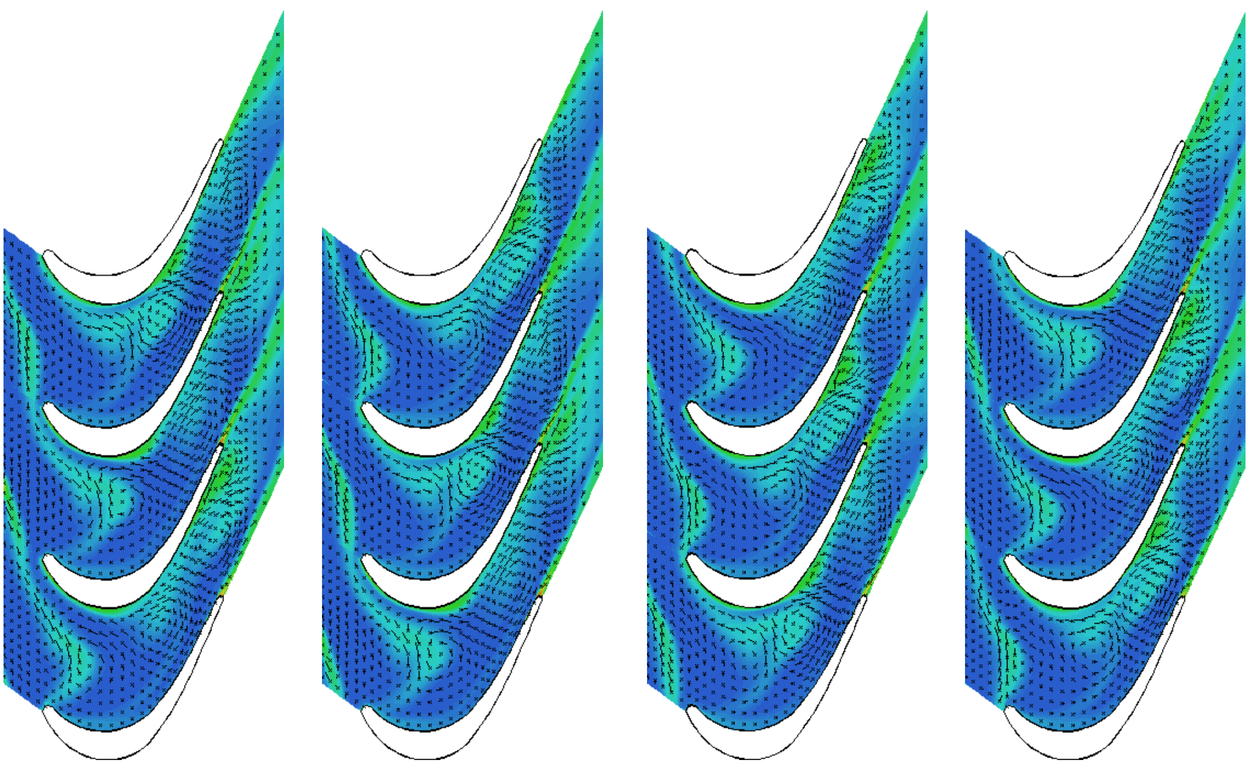
Table 1: Turbine stage geometric data

Parameter	Value
Inlet total pressure, [MPa]	0.6373
Inlet total temperature, [K]	1213
Inlet isobaric specific heat, [J/kg/K]	1200
Exit isobaric specific heat, [J/kg/K]	1156
Exit static pressure, [MPa]	0.2225
Cooling air mass flow rate, %, total mass flow rate basis	2.7
Cooling air temperature, [K]	723

Table 2: Turbine stage gasdynamic data

Numerical results

The flow computations have been performed on grids of about 400 thousand cells per one blade-to-blade passage and with $y^+ \approx 1 \div 2$ for near wall cells.

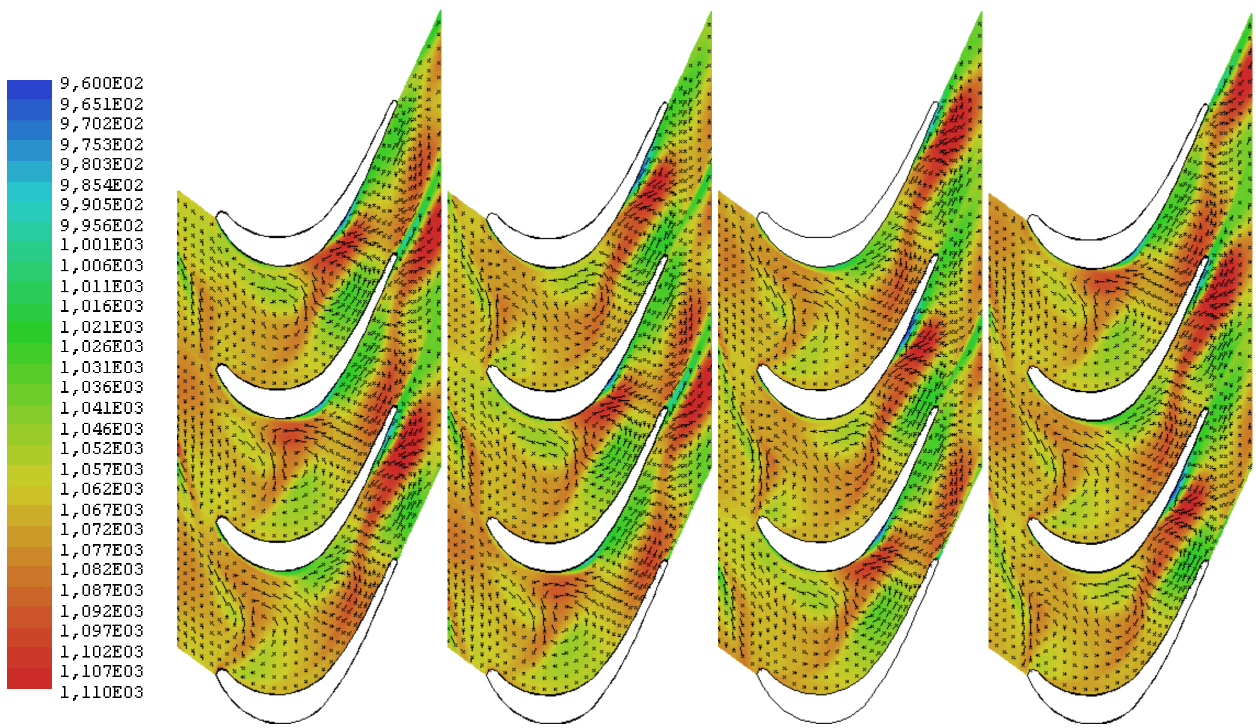


a) b) c) d)

Fig.2: Entropy function and fluctuation velocity vectors
Rotor mid section: (a) $t = 0$; (b) $t = 0.25T_2$; (c) $t = 0.5T_2$; (d) $t = 0.75T_2$

Fig. 2 shows the entropy function contours and fluctuation velocity vectors at the mid section of the rotor wheel for the four instants at unsteady flow period *T* that is equal to the time of the rotor rotation through the angular pitch of the nozzle cascade. Here the points are the cell centres where the vectors start.

The cooled viscous wakes from guide vanes are clearly seen to penetrate into the rotor cascade. Each wake after entrance in blade-to-blade passage of the rotor is cut off by the rotor blade and drifted to its suction side. Here thickening of wakes occurs through both acceleration of the main stream and formation of the vortex pairs, which are described below. The wake thickening results in neighbouring wakes joining at the suction surface of the rotor blade.



a) b) c) d)

Fig.3: Relative total temperature contours and fluctuation velocity vectors Rotor mid section: (a) $t = 0$; (b) $t = 0.25T_2$; (c) $t = 0.5T_2$; (d) $t = 0.75T_2$

It is seen from Fig. 2 that the variable parts of velocity vectors at the wake transit region are oriented along wake axis in the direction that is contrary to those of its transport. Because of mentioned effects the vortex pairs are formed. This flow behaviour is simply explained by wake interpretation as reverse jet. Shown in Fig. 3 are the relative total temperature contours for the rotor mid section. The cooled stator wakes are evidently seen in the relative frame. The total temperature field behind the stator blade is characterised not only tangential non-uniformity generated by cooled wakes but also radial one caused by decreasing mass flow rate of cooling air from root to tip. An analysis of flow pattern presented in Fig. 3 demonstrates significant amplifying non-uniformity of relative total temperature for mid tangential section of rotor wheel. Taking into account proximity of Prandtl number to 1, the present phenomenon may be explained by radial flow in wakes. In fact the radial velocity contours displayed in Fig. 4 show that the flow in the wake region is characterized by radial velocity components directed to the blade root. It is easy explained by local violation of the radial equilibrium law. On the contrary the flow drift from root to tip is observed in the flow region between two neighbouring wakes. The largest radial velocity components take place in the zones containing vortex pairs generated by the wake/cascade interaction. As a result the vortex pairs have clearly defined 3D pattern. As investigation shows the similar behaviour of unsteady cascade flow is typical not only for air-cooled blades but also for adiabatic flow conditions. The influence of three-dimensionality of interaction of stator viscous wakes and rotor wheel blades on radial distribution of energy losses and stage efficiency should be studied additionally.

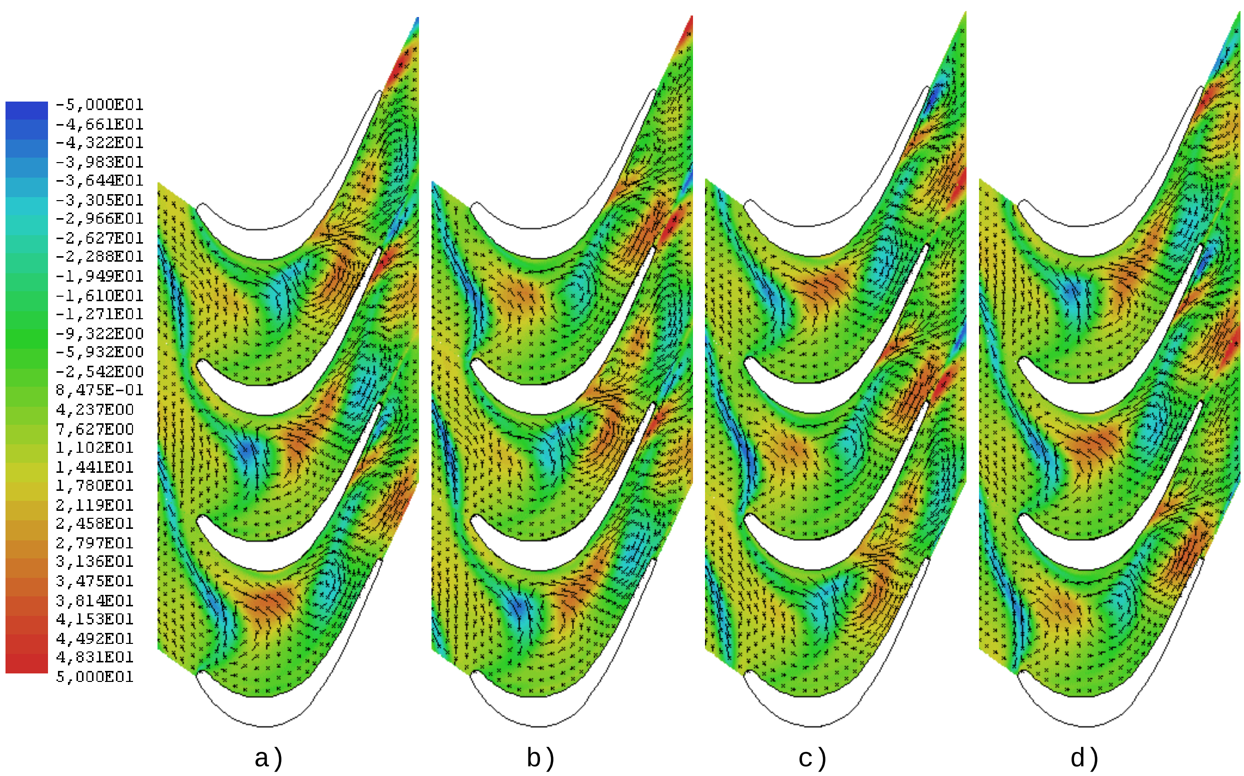


Fig.4: Radial velocity contours and fluctuation velocity vectors Rotor mid section: (a) t = 0; (b) $t = 0.25T_2$; (c) $t = 0.5T_2$; (d) $t = 0.75T_2$

The cooled wake transport through the rotor wheel leads to the considerable temporal fluctuation of temperature at the rotor blade surface as it displayed in Fig. 5 for the blade mid section. Here x is the distance from the rotor leading edge along the rotation axis and C_x is the axial chord of the rotor blade. The blade temperature minimum is located at the rotor suction surface near the vortex region between two stator wakes, travelling one after another. It is probably caused by the fact that cooled air is concentrated in the region, flowing from two neighbouring wakes in particular due to radial flow. The phenomenon can be affected by a number of factors, specifically cooled wake temperature and spanwise distribution of flow rate of cooling air.

Fig. 6 presents comparison of temperature distributions at the rotor blade surface that were obtained from steady and unsteady flow computations. Principal cause of considerable quantitative discrepancy of results presented is seemed to be essential three-dimensionality of periodically unsteady turbine stage flow, caused by stator blade wake transport through the rotor cascade.

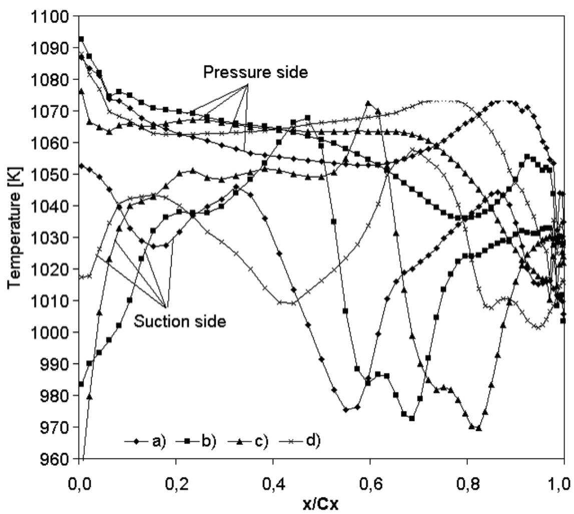


Fig.5: Blade static temperature for unsteady flow computation at rotor mid section:
(a)t=0;(b)t=0.25T_2;(c)t=0.5T_2;(d)t=0.75T_2

Summary

The computations of 3D periodically unsteady viscous flow are performed for the turbine stage. The flow unsteadiness was evoked by reciprocally moving blade rows. The effect of temperature segregation in the turbine rotor is studied for the turbine stage with air-cooled stator blades. It is shown that stator wake transport through the rotor blade-to-blade passage is characterized by clearly defined flow three-dimensionality.

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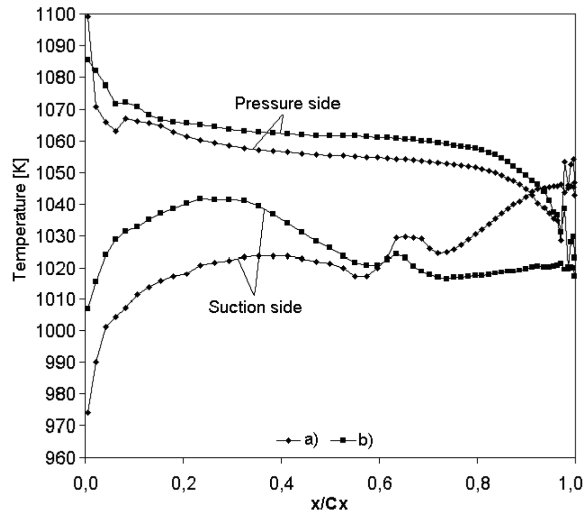

Fig.6: Blade static temperature at rotor mid section:
(a) steady flow;
(b) time-averaged unsteady flow

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